National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-3760

ST - RWP - AI — 10315

NASA TT F-9664

N65-21006 (ACCESSION NUMBER)
(THRU) 1
12 (PAGES)
(CODE) 1
(NASA CR OR TMX OR AD NUMBER)
(CATEGORY) 07
FACILITY FORM 602

ON THE PROPAGATION OF POWERFUL RADIOWAVES IN THE IONOSPHERE

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GPO PRICE $ ______

OTS PRICE(S) $ ______

Hard copy (HC) $1.00

Microfiche (MF) $0.50

8 APRIL 1965

NASA TT F-9664
~~ST - RWP - AI - 10~~ 315

ON THE PROPAGATION OF POWERFUL RADIOWAVES IN THE IONOSPHERE*

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21006

SUMMARY

The propagation of powerful radiowaves in the lower ionosphere is considered. Formulae are obtained for the amplitude of wave field intensity, as it penetrates into the feebly-ionized plasma.

* * *

The question of "auto-reaction" of powerful radiowaves in the ionosphere was worked out in detail in Gurevich works [1, 2] on the basis of joint consideration of the joint solution of nonlinear wave equation in the geometric optics approximation and of "elementary theory" expressions for the permittivity ε and conductivity σ of the plasma. The auto-reaction factor leads then, as a rule, to the necessity of numerical integration. We shall pause below at further length on the question of propagation of such waves in the lower ionosphere, whose field strength exceeds significantly a certain characteristic plasma field E_p (see [3]).

author

I. - INITIAL CORRELATIONS

From the system of Maxwellian equations we obtain in the usual fashion the equation for the electric field of the wave :

$$\nabla^2 E - \operatorname{grad} \operatorname{div} E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial j_t}{\partial t}. \quad (1)$$

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* K voprosu o rasprostranenii sil'nykh radiovoln v ionosfere

In the equation (1), the total current vector is

$$j_t = e \int v f(t, r, v) dv, \qquad (2)$$

where e is the charge of the electron, $f(t, r, v)$ is the distribution function of electrons, satisfying the Boltzmann equation.

For the radioband in the ionosphere, the time $\tau_E \sim 1/\omega$, during which the wave field varies substantially, is, as a rule, much lesser than the relaxation time of electron energy $\tau_p \sim 1/\delta\nu$ (here ν is the collision frequency of the electron with heavy plasma particles, δ is the fraction of electron energy lost by it during one collision). This means that the medium's parameter does not have the time to vary as rapidly as does the wave field. The distribution function settles at a certain value of f_0, independent from the time, of which the deflection amplitudes are small (of the order of δ and $\delta\nu/\omega$).

For a feebly-ionized uniform plasma, that is for a plasma, where the collision frequency of the electron is mainly determined by collisions with molecules, we may write for the f_0 distribution function in the presence of a high-frequency field $E = E_0 e^{i\omega t}$ (see [4]) :

$$f_0 = C \exp\left\{ - \int^{v} \frac{mv\,dv}{kT + (e/3\nu\delta)\,\mathrm{Re}(E^* u)} \right\}. \qquad (3)$$

Here C is the normalization constant, k is the Boltzmann constant, T is the temperature of plasma's heavy particles, m is the mass of the electron, u is the directed motion velocity of electrons, determined by the equation

$$\frac{\partial u}{\partial t} + \nu(v) u = \frac{eE}{m} + \frac{e}{mc}[uH^0], \qquad (4)$$

where H^0 is the constant magnetic field of the Earth.

At arbitrary polarization of the field E, we obtain by its expansion along three basic polarization directions :

$$\mathrm{Re}(E^* u) = \frac{e\nu}{m}\left[\frac{E_\parallel^2}{\omega^2 + \nu^2} + \frac{2(E_\perp^+)^2}{(\omega + \omega_H)^2 + \nu^2} + \frac{2(E_\perp^-)^2}{(\omega - \omega_H)^2 + \nu^2}\right], \qquad (5)$$

where $E_\parallel$ is the plane polarized field with $E_\parallel \parallel H^0$, $E_\perp^+$, $E_\perp^-$ are fields circularly polarized in a plane perpendicular to H^0, and rotating in a direction coinciding (—) and (+) opposite to the direction of electron rotation in the magnetic field, $\omega_H = |e|H^0/mc$ is the gyrofrequency. For a given polarization of the field, the expression

$$f_0 = C\exp\left\{-\int_0^x \frac{x\,dx}{1+\gamma^2(\varphi^2+\alpha^2x^2)^{-1}}\right\}, \qquad (6)$$

in which $x = v/\beta$, $\beta^2 = kT/m$ is valid for f_0. For the case of polarization in a plane perpendicular to

$$\gamma^2 = \frac{2e^2E_0^2}{3m^2\,\delta\omega^2\beta^2} = \frac{2e^2E_0^2}{3m\,\delta\omega^2kT}, \quad \alpha^2 = \frac{(\pi a^2N_m)^2\beta^2}{\omega^2}, \quad \varphi^2 = \frac{(\omega\pm\omega_H)^2}{\omega^2}, \qquad (7)$$

for the case of plane polarized field, γ^2 will decrease by a factor of 2 and φ^2 will become equal to unity.

When writing the correlation (6), it was taken into account that in the lower ionosphere

$$\nu(v) = \pi a^2N_m v, \quad \pi a^2 = 4{,}4\cdot 10^{-16}\ \text{см}^2$$

(here N_m is the concentration of molecules). The function of losses $\delta(v)$ for the lower ionosphere may be considered as approximately constant and equal to $1.6\cdot 10^{-3}$ (see [5]).

Effecting the integration in (6), we obtain

$$f_0 = C\exp\left\{-\frac{x^2}{2}+\frac{\gamma^2}{2\alpha^2}\ln\left(1+\frac{\alpha^2x^2}{\varphi^2+\gamma^2}\right)\right\} = C\left(1+\frac{\alpha^2x^2}{\varphi^2+\gamma^2}\right)^{\gamma^2/2\alpha^2}\times\exp\left\{-\frac{x^2}{2}\right\}. \quad (8)$$

It follows from this expression, that on the condition

$$\gamma^2 \gg \varphi^2 \qquad (9)$$

(which is equivalent to the condition $E \gg E_p$) the representation of the distribution function in the form

$$f_0 = C\exp\left\{-\frac{\varphi^2}{2\gamma^2}x^2 - \frac{\alpha^2}{4\gamma^2}x^4\right\}. \qquad (10)$$

is quite admissible through velocities $v \leqslant \beta\gamma/\alpha$.

The brought up value of f_0 can be applied with sufficient precision for the problems, where quantities, averaged by the entire range of velocities are considered.

Indeed, considering the distribution function (10), we notice that it is essentially different from zero in the velocity range from 0 to $v_c = x_c\beta$, where

$$x_c^2 = \frac{-\varphi^2 + \sqrt{\varphi^4 + 4\gamma^2\alpha^2}}{\alpha^2}; \qquad (11)$$

for velocities $v > v_c$ the distribution function sharply (exponentially) drops. But at condition (9), the limit velocity $v_c = x_c\beta$, to which the representation of f_0 in the form (10) is still valid, is substantially greater than v_c.

Inasmuch as **u** and f_0 in (1) depend on **E**, the equation (1) is a nonlinear integro-differential equation relative to wave's electric field strength. In the general case the parameters of the medium depend on time, by the strength of what overtones will occur at propagation of a wave with the frequency ω. However, in the first approximation relative to δ and $\delta\nu/\omega$, the main harmonic of the electric field **E**, satisfying the equation (1), may be separated, but with the essential difference that f_0 in the expression for j_t depends only on amplitude of **E** (see [2]). The overtones of the field **E** have an amplitude of the order $\frac{\delta\nu}{\omega}$.

We shall limit ourselves in the following to consideration of particular cases of wave polarization:

a) a plane polarized wave with a vector of **E**, coinciding in direction with the constant magnetic field H^0 (transverse propagation, axis $z \perp H^0$);

b) a circularly polarized wave in a plane, perpendicular to H^0 (longitudinal propagation, axis $z \parallel H^0$). In the last case, the propagation of an ordinary and extraordinary wave will be considered separately.

In these cases the system (1) is reduced to a single equation

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2}\varepsilon'(|E|, z)E = 0 \qquad (12)$$

with the boundary condition $E(z = 0) = E_0$ ($z = 0$ is the plasma boundary). Here $\varepsilon'(|E|, z) = \varepsilon - i4\pi\sigma/\omega = (n - i\varkappa)^2$ is the complex dielectric constant, **n** is the index of refraction, $\varkappa$ is the indicator of absorption (absorption coefficient).

Utilizing the correlation (see [3])

$$j_t = \sigma E + i\omega\frac{\varepsilon-1}{4\pi}E = -\frac{4\pi e}{3}\int_0^\infty uv^3\frac{\partial f_0}{\partial v}dv, \qquad (13)$$

we obtain for ε'

$$\varepsilon'(|E|, z) = 1 + \frac{4\pi\beta^3}{3N}\frac{\omega_0^2}{\omega^2}\int_0^\infty \frac{\varphi + i\alpha x}{\varphi^2 + \alpha^2 x^2}x^3\frac{\partial f_0}{\partial x}dx, \qquad (14)$$

where N is the concentration of electrons, $\omega_0^2 = 4\pi e^2 N/m$ is the plasma frequency. Utilizing the distribution function (10), the expression (14) may be somewhat simplified :

$$\varepsilon'(|E|, z) = 1 - \frac{4\pi\beta^8}{3N\gamma^2}\frac{\omega_0^2}{\omega^2}\int_0^\infty (\varphi + i\alpha x)x^4 f_0 dx. \qquad (15)$$

If the properties of the medium vary sufficiently slowly, the solution of the equation (12) in the geometric optics approximation can be written :

$$E = \frac{E_0}{\sqrt[4]{\varepsilon'(|E|, z)}}\exp\left\{-i\frac{\omega}{c}\int_0^z \sqrt{\varepsilon'(|E|, z)}\,dz\right\}. \qquad (16)$$

Here we limited ourselves to the wave propagating along the axis z. Besides the standard condition of geometric optics approximation (see [6])

$$\frac{\lambda}{4\pi(\varepsilon')^{3/2}}\frac{d\varepsilon'}{dz} \ll 1$$

in the case of propagation of a powerful wave the condition $\varkappa/n \ll 1$, must be satisfied, that is, the amplitude of the field E must vary little on the wavelength.

It should be noted also, that for the computation of $\varepsilon'(|E|, z)$ the distribution function f_0, which is the solution of the kinetic equation for a uniform plasma, was utilized. When considering an inhomogenous medium, it is assumed that the properties of the medium are localized, that is, the density of the current j_t at the given point is determined by the field at the same point. For the case of intense fields this is valid, provided the amplitude of the field varies little over the length of relaxation for the energy $l/\sqrt{\delta}$ (l being the length of the free path of the electron in the plasma).

2. — "AUTO-REACTION" EFFECT OF A POWERFUL WAVE

Exploiting the circumstance, that $\varepsilon'(|E|, z)$ depends only on $|E|$, we have from the equation (16) for the field amplitude

$$|E| = E_0 \exp\left\{-\frac{\omega}{c}\int_0^z \varkappa(|E|, z)\,dz\right\}. \qquad (17)$$

Here we neglected the influence of the multiplier $(\varepsilon')^{-1/4}$ on absorption. The methods of numerical integration of the equation (17) are developed in the reference [1]. However, two boundary conditions can be outlined, for which this equation has a solution in an explicit form

$$1) \quad \varphi^4 \ll 4\gamma^2\alpha^2. \qquad (18)$$

This case is comparatively easily realized for the extraordinary wave in the frequencies ω, close to ω_H; at the same time, the condition (18) is in this case the gyroresonance condition de facto. For the ordinary wave and in case of transverse propagation, the observance of condition (18) makes high field intensities prerequisite, and so much the greater that the wave frequency is higher. At the same time, it should be borne in mind that in the case when the main role is played by collisions with molecules, the following condition should be satisfied [4]

$$\delta(\gamma\alpha)^{1/2} \ll 1. \qquad (19)$$

For the distribution function we may write

$$f_0 = \frac{N\beta^{-3}}{\pi\Gamma(3/4)}\left(\frac{\alpha}{2\gamma}\right)^{3/2}\exp\left\{-\frac{\alpha^2 x^4}{4\gamma^2}\right\}. \qquad (20)$$

Substituting (20) into the expression for the complex dielectric constant (15), we obtain

$$\varepsilon'(|E|, z) = 1 - 0{,}49\frac{\varphi}{\gamma\alpha}\frac{\omega_0^2}{\omega^2} - i\,0{,}68\frac{1}{(\gamma\alpha)^{1/2}}\frac{\omega_0^2}{\omega^2}. \qquad (21)$$

Hence, it follows that in the considered case the condition of geometric approximation is disrupted in the region, where the imaginary part of the expression (21) is of the order of the unity (at the same time the condition of geometrical optics approximation's applicability for a powerful wave is disrupted).

Below the reflection region, that is at $0{,}68\,(\gamma\alpha)^{-1/2}\omega_0^2\omega^{-2} \ll 1$, the absorption factor and the refractive index have the form

$$n = 1 + 0{,}1\,\frac{\omega_0^2}{\omega^2}\,\frac{1}{\gamma\alpha}\left(\frac{\omega_0^2}{\omega^2} - 4{,}25\,\varphi\right), \quad \varkappa = 0{,}34\,\frac{1}{(\gamma\alpha)^{1/2}}\,\frac{\omega_0^2}{\omega^2}. \tag{22}$$

$$2)\quad \varphi^4 \gg 4\,\gamma^2\alpha^2. \tag{23}$$

This case is easily realized for frequencies $\omega \gg \omega_H$. At $\omega_H \approx \omega$ the condition (23) is poorly observed in the lower part of the ionosphere. It is clear, that for the extraordinary wave the condition (23) is not fulfilled at $\omega \simeq \omega_H$ In the considered case

$$f_0 = N\,\beta^{-3}\left(\frac{\varphi^2}{2\,\pi\gamma^2}\right)^{3/2}\exp\left\{-\frac{\varphi^2}{2\,\gamma^2}\,x^2\right\}; \tag{24}$$

$$\varepsilon'(|E|,\,z) = 1 - \frac{1}{\varphi}\,\frac{\omega_0^2}{\omega^2} - i\,\frac{2{,}14}{\varphi^3}\,\gamma\alpha\,\frac{\omega_0^2}{\omega^2}. \tag{25}$$

The reflection region is determined by the condition $\omega_0^2/\varphi\omega^2 \sim 1$. Below it, that is at $\omega_0^2/\varphi\omega^2 \ll 1$, the refractive index and the absorption factor are

$$n \simeq 1 - \frac{1}{\varphi}\,\frac{\omega_0^2}{\omega^2}, \quad \varkappa \simeq \frac{\omega_0^2}{\omega^2}\,\frac{\gamma\alpha}{\varphi_3}. \tag{26}$$

As already noted above, the solution of the equation (17) may be written in an explicit form in the two considered boundary cases. To that effect we shall take advantage of the circumstance, that the dependence of $\varkappa$ on γ has the form

$$\varkappa = \varphi(z)\gamma^y, \tag{27}$$

where $\varphi(z) = 0{,}34\,\omega_0^2\omega^{-2}\alpha^{-1/2}$, $y = -1/2$ for the first case (18), and $\varphi(z) = \omega_0^2\alpha/\omega^2\varphi^3$, $y = 1$ for the second case (23).

For the dependence of $\varkappa$ on γ of the form (27) the solution of the equation (17) is

$$\gamma(z) = \gamma_0\left[1 + y\,\gamma_0^y\,\frac{\omega}{c}\int_0^z \varphi(z)\,dz\right]^{-1/y}. \tag{28}$$

In the first boundary condition

$$\gamma(z) = \gamma_0\left(1 - \frac{0,17}{\omega c}\ \gamma_0^{-1/2}\int\limits_0^z \omega_0^2\alpha^{-1/2}\,dz\right)^2; \tag{29}$$

and in thesecond one

$$\gamma(z) = \gamma_0\left(1 + \frac{\gamma_0}{\omega c\varphi^3}\int\limits_0^z \omega_0^2\alpha\,dz\right)^{-1}. \tag{30}$$

Note, that the representation of the solution of the equation (17) in the form (29), (30), that through a dimensionless parameter γ, is only possible for a constant value of β along the path of wave propagation. In the opposite case it is necessary to rewrite the correlation (28) directly for the wave amplitude:

$$E(z) = E_0\left[1 + y\gamma_0^y\frac{\omega}{c}\int\limits_0^z \varphi(z)\,dz\right]^{-1/y} \tag{31}$$

For radiowaves of sufficiently low frequencies (the condition $\delta\nu/\omega \ll 1$ being observed) the application of the obtained correlations for the calculation of wave damping is linked with the requirement of joining them in a certain transitional region. For low ionosphere heights the fulfillment of the boundary condition (18) is not only possible for the extraordinary wave with a frequency near the gyrofrequency, but also for the ordinary wave and also in the case of transverse propagation of the wave. As it penetrates the ionosphere $\gamma\alpha$ drops and, beginning from certain heights, the second boundary condition becomes workable (23).

The estimates, using directly the distribution function (10), indicate that the joining introduces the least of errors in the region where $\gamma^2 \simeq \varphi^2$, when the utilization of correlations (29) — (30) provieds an inaccuracy of the order of 25 ÷ 30 percent If the transitional region is sufficiently large, the discrepancy may be greater because of error accruing. However, it may be shown by using the expression (11), that the transitional region is of the order of several wavelengths (in any case for the values of γ and φ in Fig. 1). The relative error $\gamma(z)$ at utilization of (30) (at the expense in the determination of γ_0 in the transitional region remains approximately constant as the wave penetrates into the plasma :

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$$\frac{\Delta\gamma(z)}{\gamma(z)} \simeq \frac{\Delta\gamma_0}{\gamma_0}.$$

For the specific model of the ionosphere represented in Table 1 (data borrowed from [7]) the dependence E (z) was computed for various frequencies.

TABLE 1

z (км)	60	64	68	72	76	80	84	88	92
$N_m \cdot 10^{-14}$	65	44	30	14	7	3,5	2,5	1,8	1,2
$N \cdot 10^{-2}$	0,05	0,26	0,87	1,7	2,88	4,5	8,05	34,7	250

The results of computations are plotted in Fig. 1. The solid lines refer to the ordinary wave (curves 1, 2, 3, 4 correspond to frequencies ω_H, $2\omega_H$, $5\omega_H$, $10\omega_H$), the dashed lines refer to the extraordinary wave (frequencies ω_H, $5\omega_H$, $10\omega_H$). respond to curves 1', 3', 4'). For waves of any frequency the intensity of the field E_0 at plasma boundary is chosen the same.

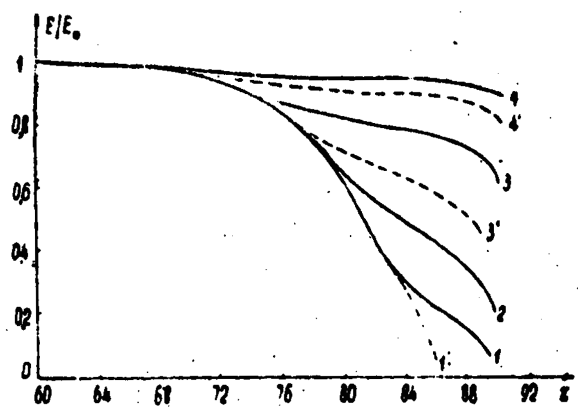

Fig. 1. Wave field amplitude dependence E (z)

Analysing the correlations (29) — (30) and the curves of Fig. 1, we may reach the following conclusions :

1) At the condition $\varphi^2 < \gamma\alpha$ the frequency dependence of absorption is very weakly expressed; for the ordinary and extraordinary waves there takes place a peculiar degeneration — they differ only by polarization (opposite rotation of the field vector).

2) At the same condition $\varphi^2 > \gamma\alpha$, the penetrating capability of the wave (ratio E/E_0) increases with the rise of E_0.

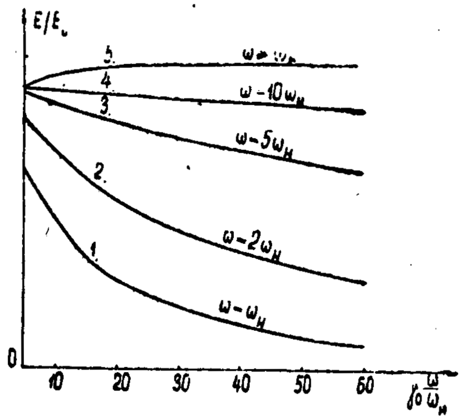

Fig. 2. - Schematic curves of E/E_0 dependence on E_0 for 2 cases : 1) $\varphi^2 > \gamma\alpha$ (curves 1, 2, 3, 4, the frequencies being indicated curves), 2) $\varphi^2 < \gamma\alpha$ (curve 5).

3) At the condition $\varphi^2 < \gamma\alpha$ the penetrating capability of the wave drops with the increase of E_0 but increases with rise of frequency (see Fig. 2).

4) At the very same condition $\varphi^2 > \gamma\alpha$, , that is when the frequency dependence of wave absorption is expressed sufficiently clearly, the curves of Fig. 1 confirm also such usual consequence of radiowave propagation as are the sharper character of the absorption of the extraordinary wave by comparison with the ordinary one, the decrease in wave absorption and in the difference in the propagation of ordinary and extraordinary waves with the increase of frequency.

In conclusion, the author expresses his deep gratitude to I. M. Vilenskiy for discussing the current work, the valuable comments and remarks, and also to O. M. Grkhov for conducting computations and constructing the graphs.

**** THE END ****

Contract No. NAS-5-3760

Consultants and Designers, Inc.
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Translated by ANDRE L. BRICHANT
on 6 and 7 April 1965

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Received on 15 January 1964.

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